Computing Positively Weighted Straight Skeletons of Simple Polygons Using an Induced Line Arrangement

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Alicante, June 2017

Related Work & Overview

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- The straight skeleton was introduced by Aichholzer et al. 1995 [1].
- Eppstein and Erickson [3] introduced in 1999 an algorithm with the current best worst-case complexity: For a simple polygon (with holes) it requires $\mathcal{O}(n^{17/11+\varepsilon})$ time and space to compute the (weighted) straight skeleton.
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- Our work is based on the work of Huber and Held (IJCGA 2012) on straight-skeleton computation based on motorcycle graphs [4].
- Using an *extended wavefront* they transform split events into edge events, shifting the complexity to another event.
- We revisit their work and show required changes to apply their approach to the weighted scenario.

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- Consists only of straight line segments.
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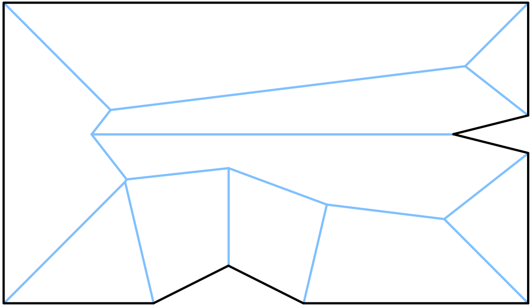
  - The vertices of the *wavefront polygons* trace out *arcs*.
  - Two events: edge event and split event



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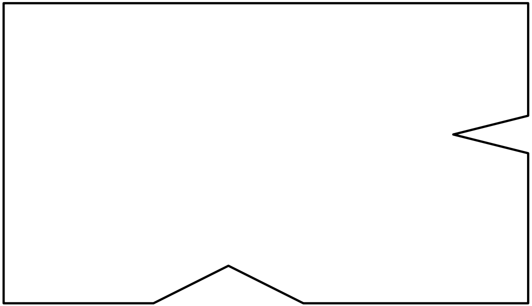

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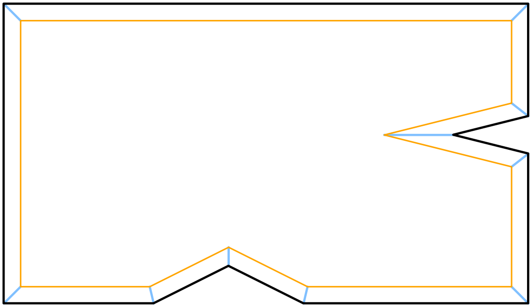

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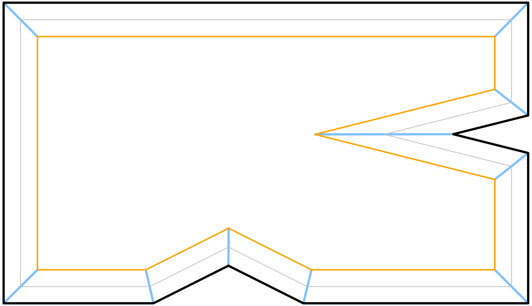

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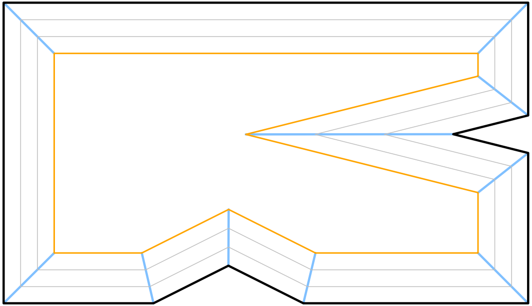

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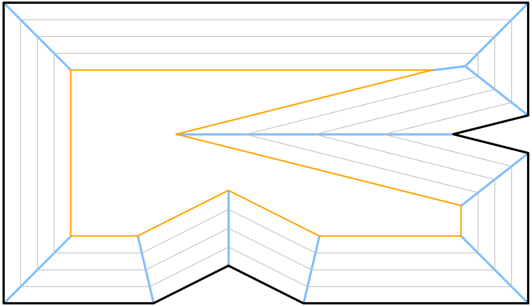

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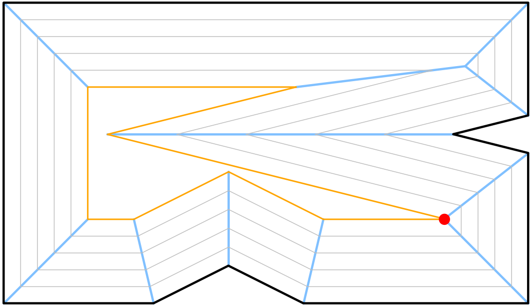

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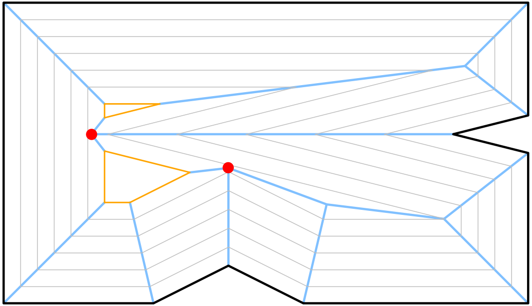

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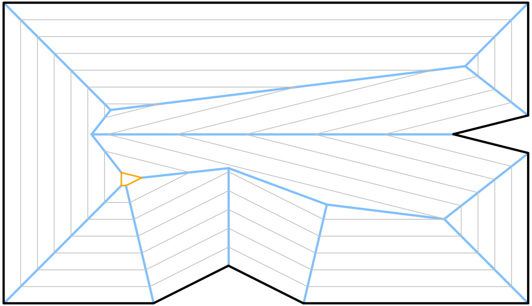

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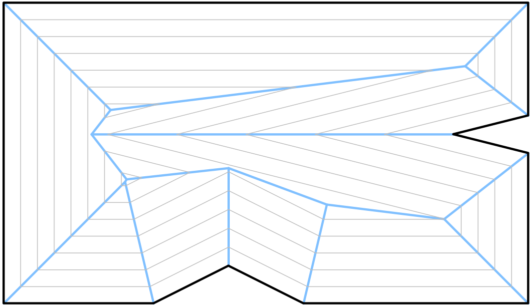

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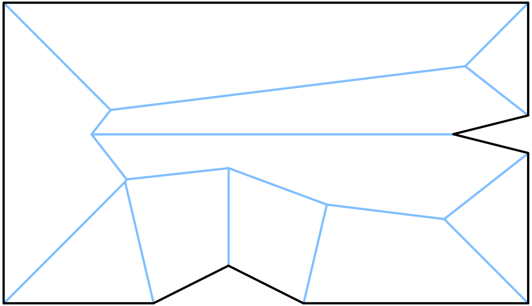
  - The *straight skeleton* consists of the blue arcs.



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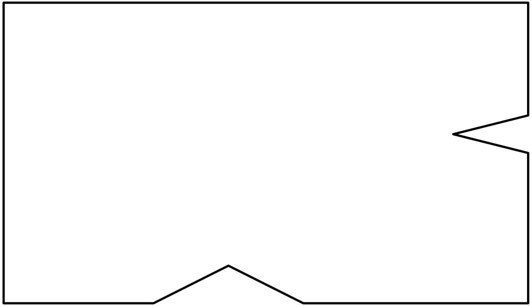
Preliminaries

- Let P a simple polygon with n vertices, r of which are reflex.
- Then $\mathcal{W}_P(t)$ is the wavefront of P at time t.
- After all components of $\mathcal{W}_P(t)$ have vanished we call the traces of the vertices of $\mathcal{W}_P(t)$ over the time-span of the propagation straight skeleton $\mathcal{S}(P)$.



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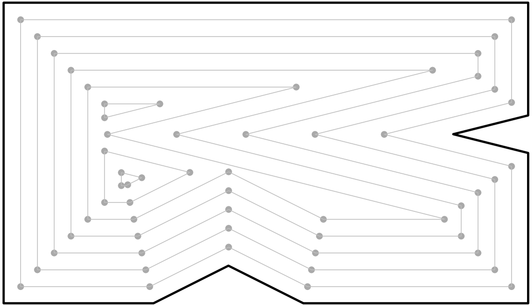

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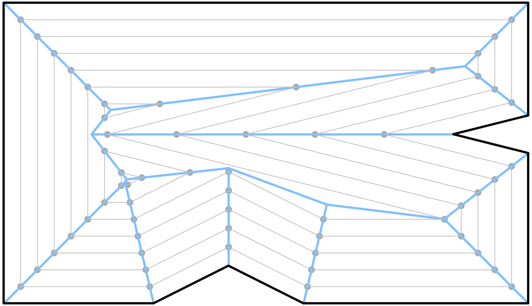
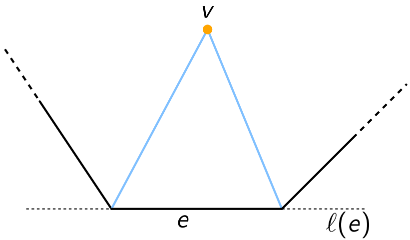


Straight Skeletons – Finding Events

Edge Events

- Compute vanishing time for every edge if finite.
- Enqueue all events in priority queue.
- $\mathcal{O}(n \log n)$ time and $\mathcal{O}(n)$ space.



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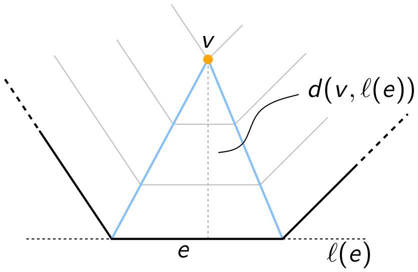

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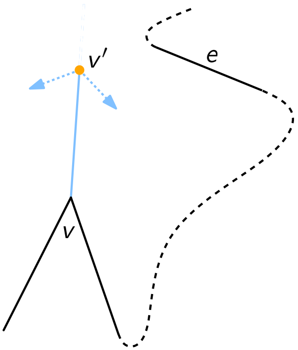

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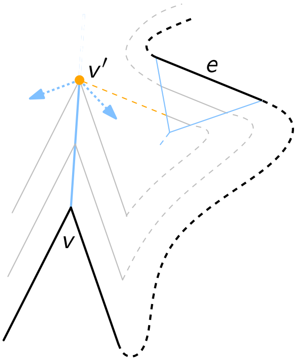

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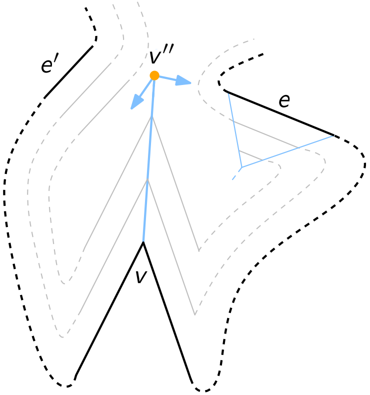

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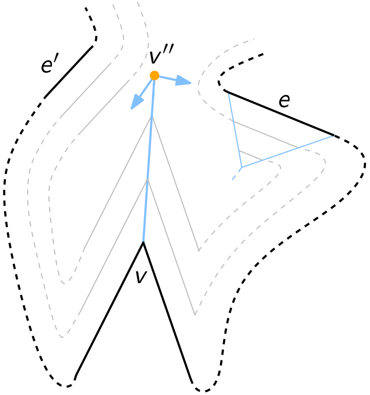

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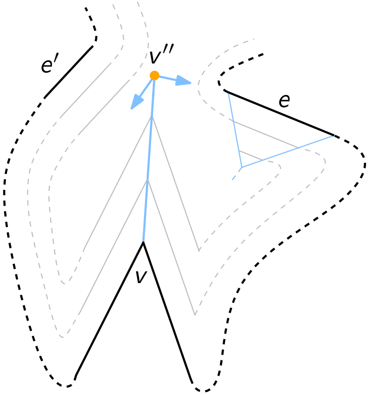
Multi-Split Event

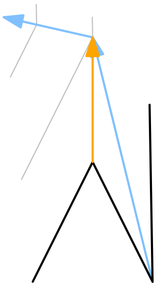
- When two reflex wavefront vertices meet at a common point[2].



[2] Related to vertex event [3].

Tracing Reflex Arcs

- Every event, where a reflex wavefront vertex is involved, reduces the number of reflex vertices in $\mathcal{W}_P(t)$.
- *Can we know these reflex arcs in advance?*



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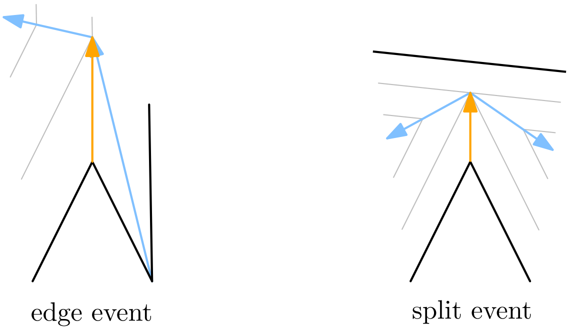

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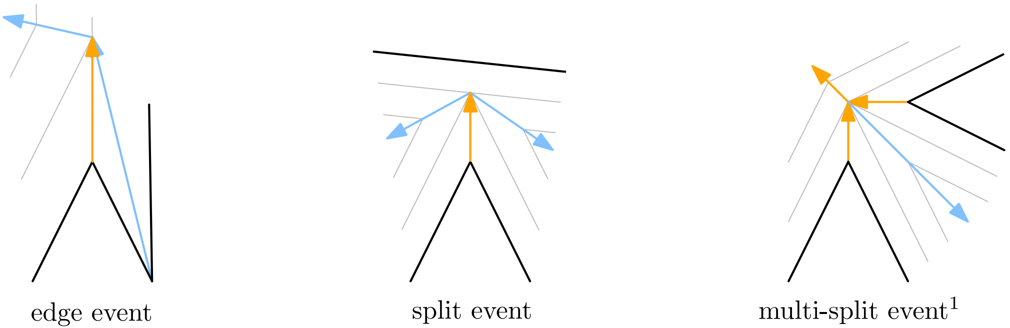

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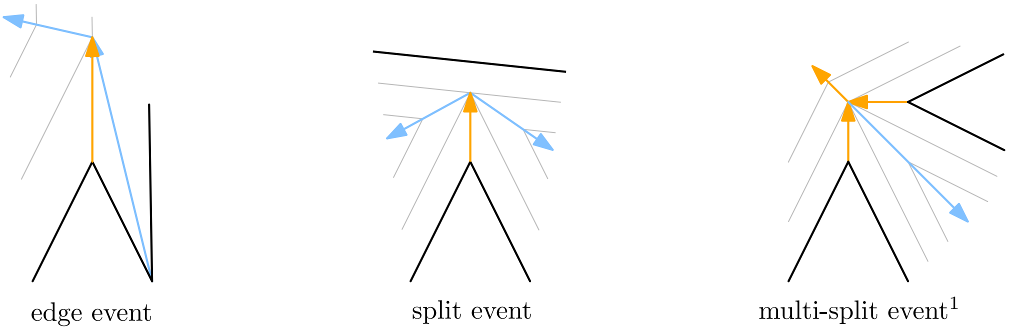

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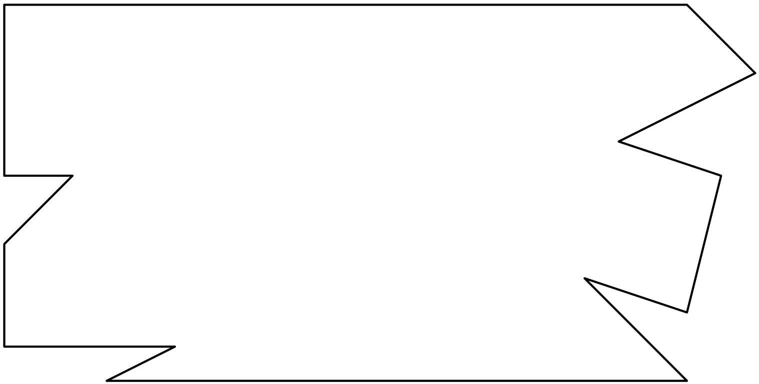
Motorcycle Graph and $\mathcal{S}(P)$

- Start a motorcycle m for every reflex vertex v of $\mathcal{W}_P(t)$ such that m inherits the velocity vector of v.
- Every motorcycle leaves a trace behind and stops if it crashes into another trace or the polygon boundary.
- The *motorcycle graph* $\mathcal{M}(P)$[3] is formed when all motorcycles have crashed.



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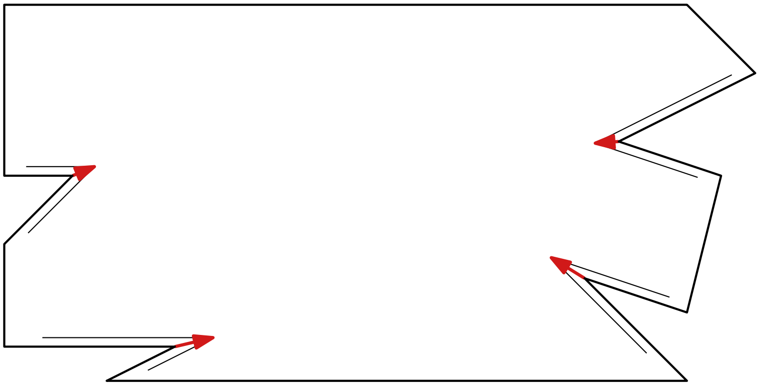

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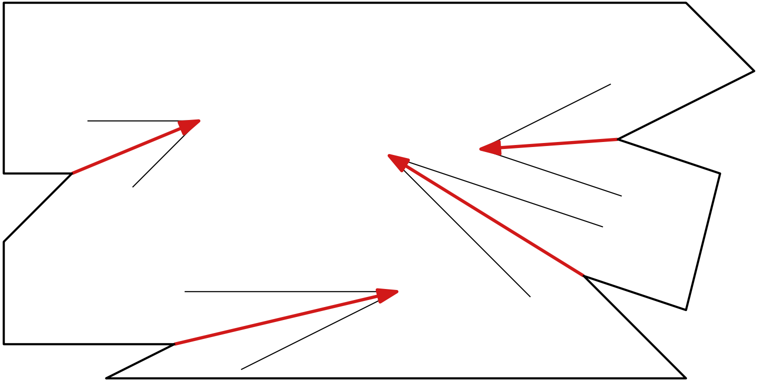

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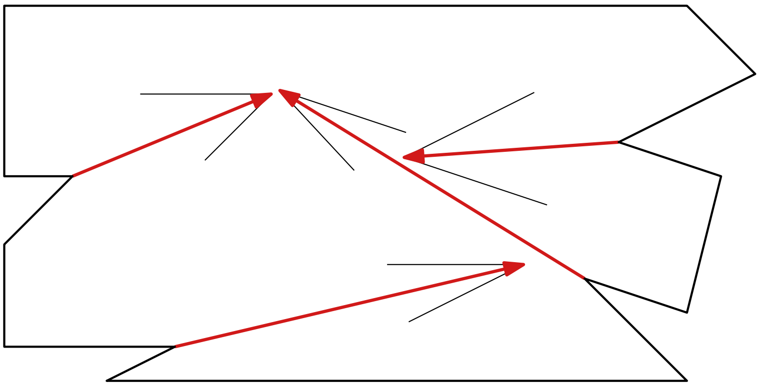

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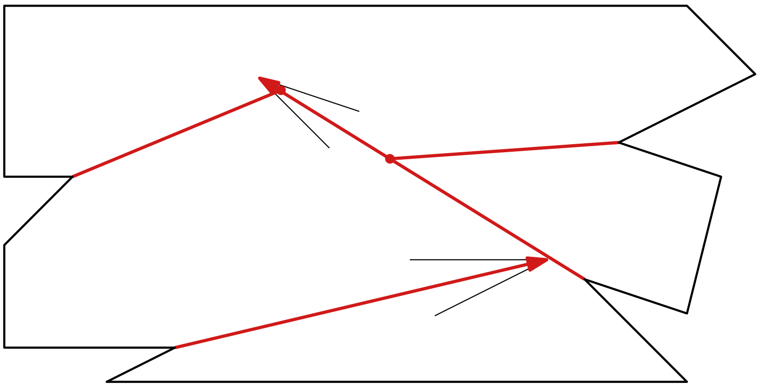

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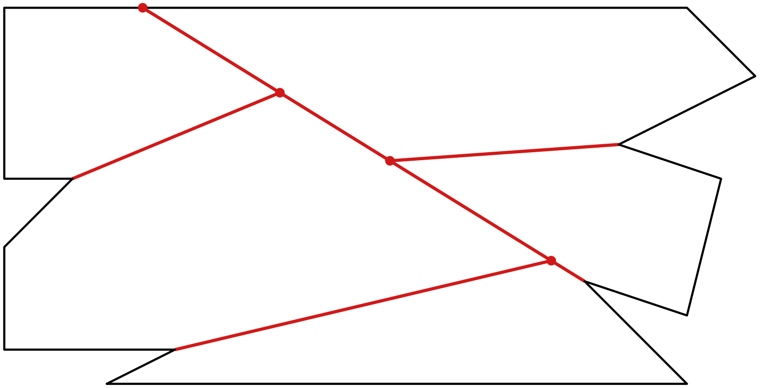

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Motorcycle Graph and $\mathcal{S}(P)$ (cont'd)

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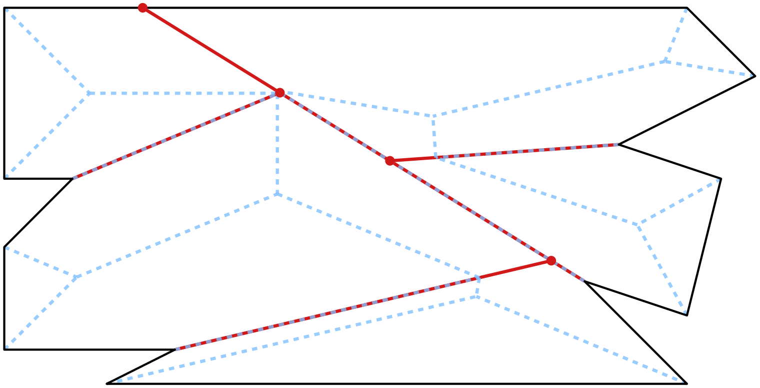

[4] Various publications [2, 3]

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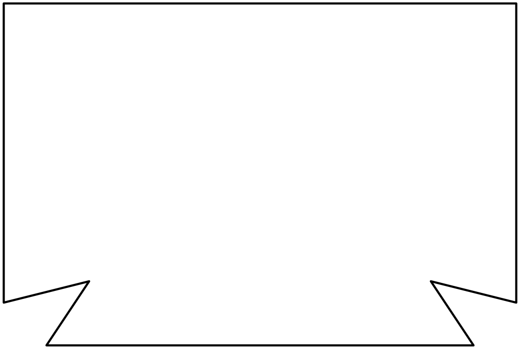

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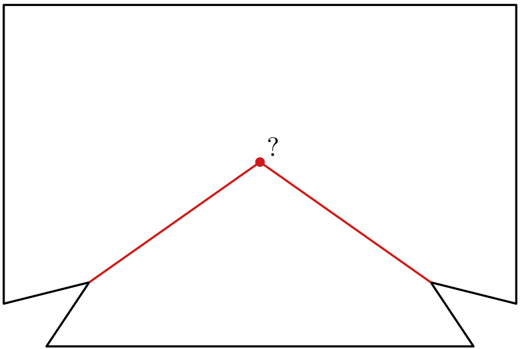

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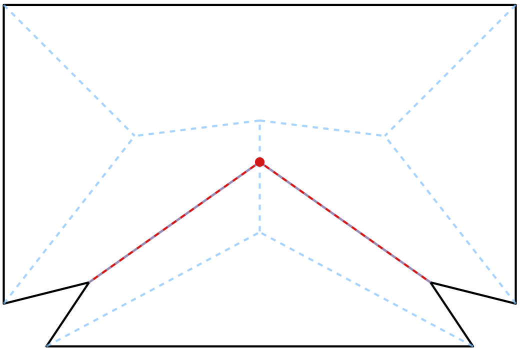

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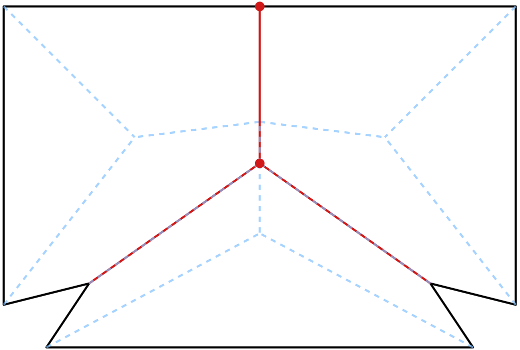

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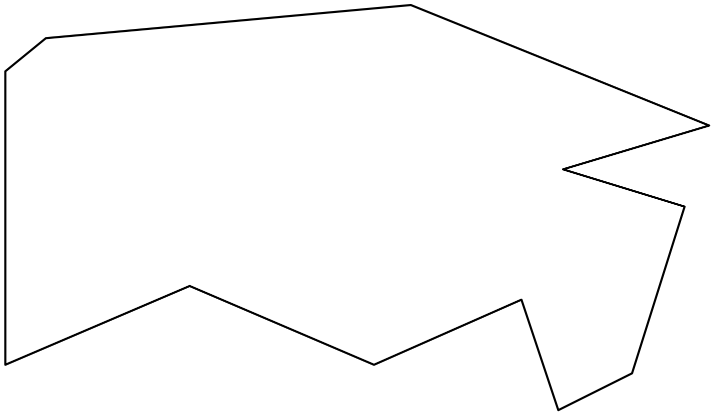
Extended Wavefront $\mathcal{W}_P^*$

- Huber and Held introduce the *extended* wavefront $\mathcal{W}_P^*(t)$.
- $\mathcal{W}_P^*(t)$ consists of the wavefront at time t and the portion of $\mathcal{M}(P)$ that lies inside that wavefront.
- All regions in $\mathcal{W}_P^*(t)$ are convex and all events are edge events.



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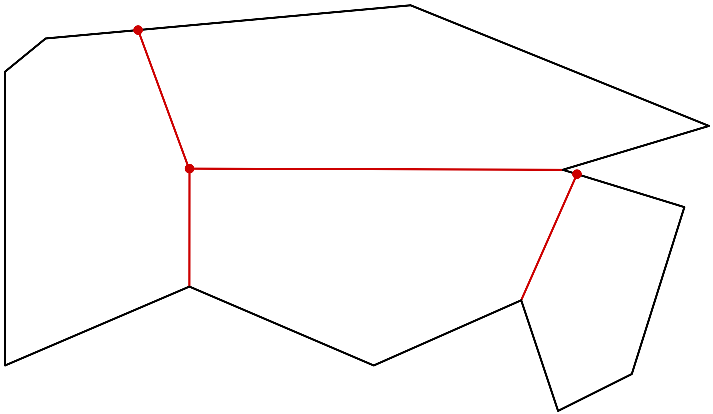

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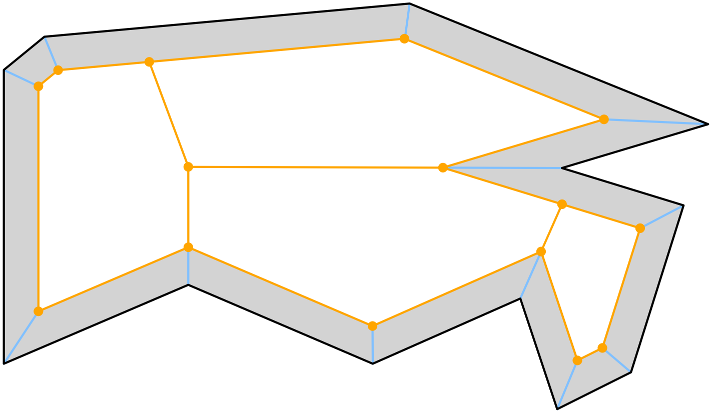

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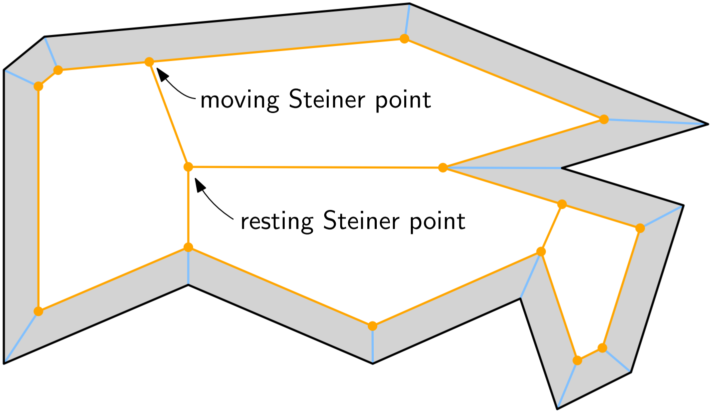

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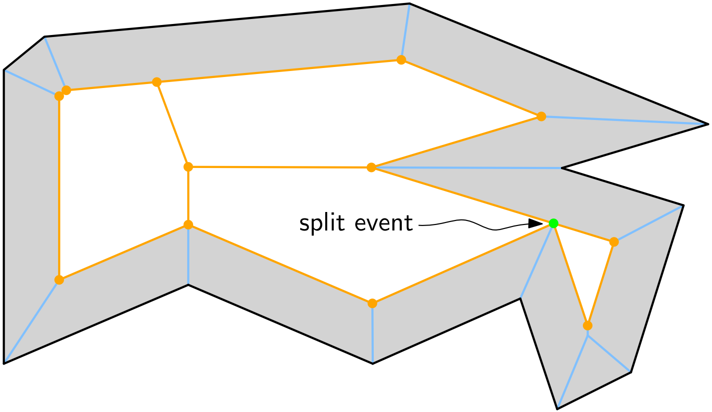

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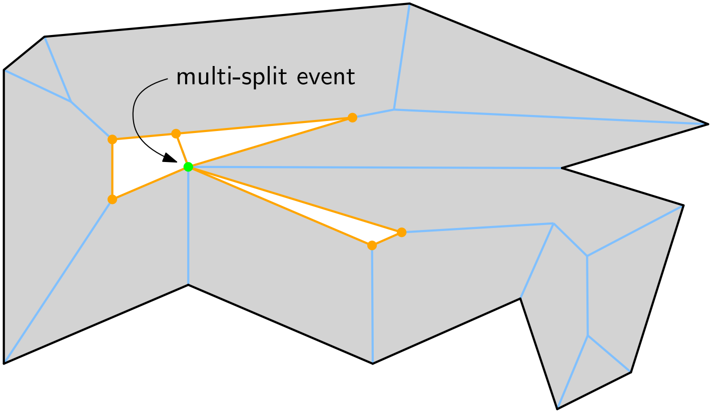

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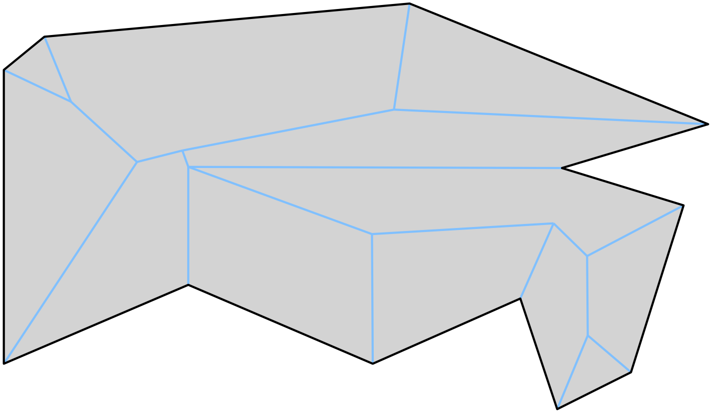

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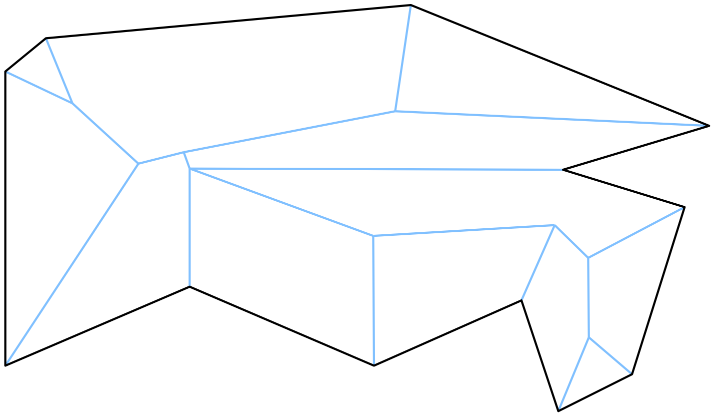

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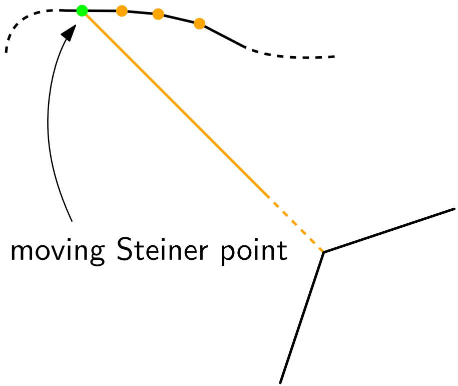

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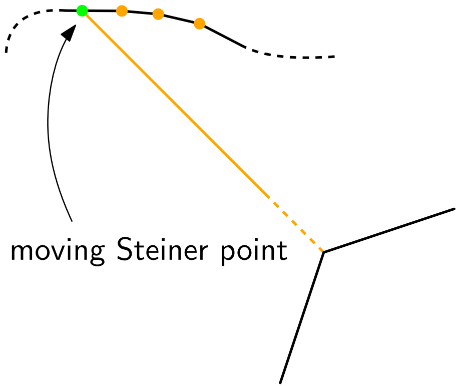

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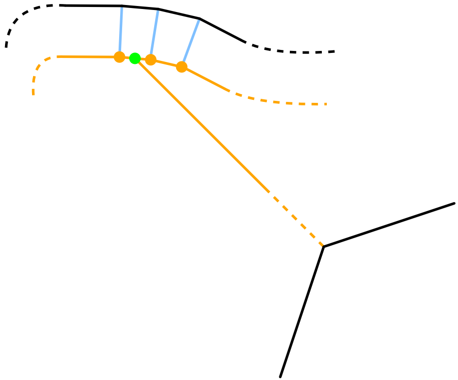

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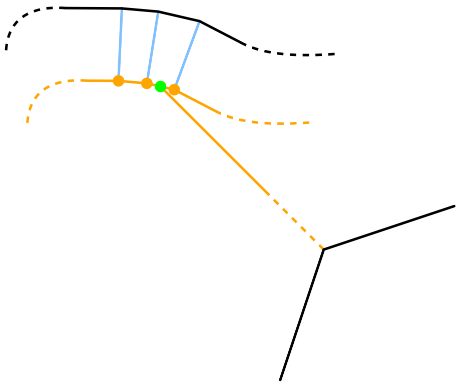

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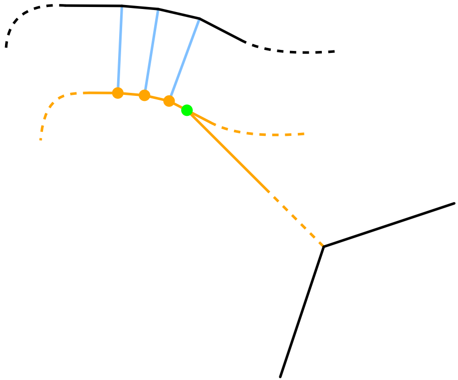
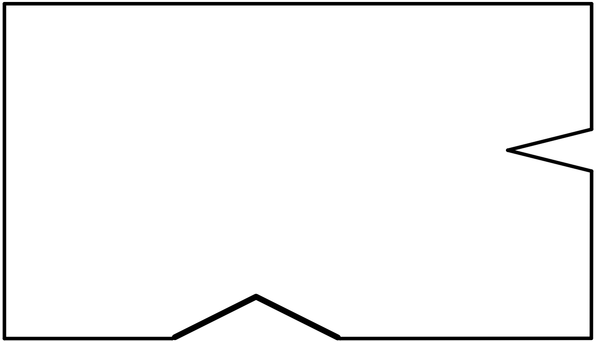


Weighted Straight Skeleton $\mathcal{S}(P, \sigma)$

- Wavefront $\mathcal{W}_P(t, \sigma)$ traces out the weighted straight skeleton $\mathcal{S}(P, \sigma)$, s.t. the weight function $\sigma \in \mathbb{R}^+$ provides the strictly positive edge weights.
- Thick edges have σ of about 3, unit weight otherwise.
- $\mathcal{S}(P)$ (left) and $\mathcal{S}(P, \sigma)$ (right).



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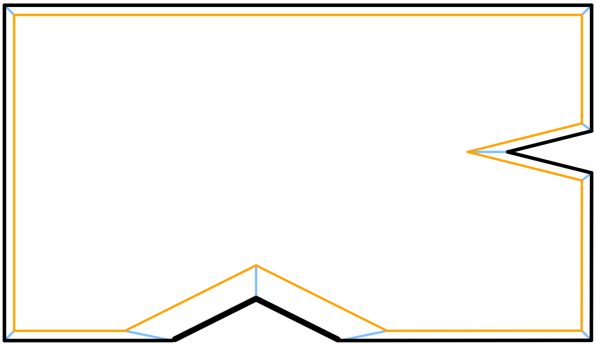

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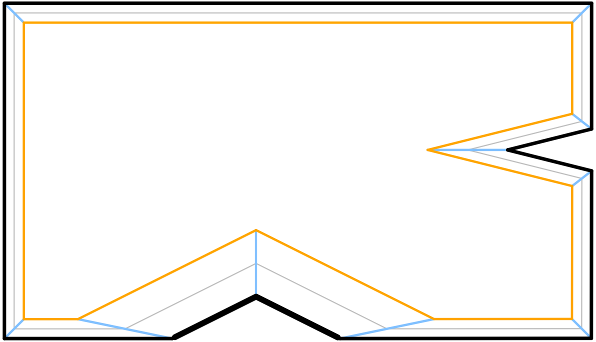

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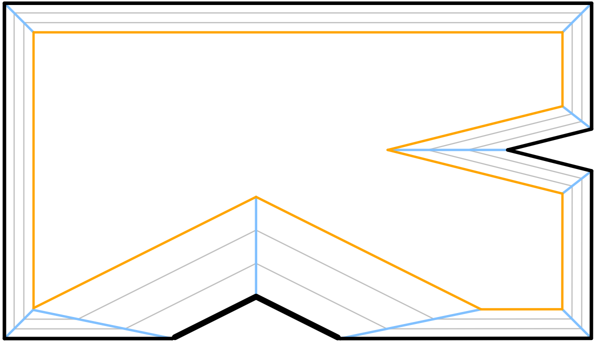

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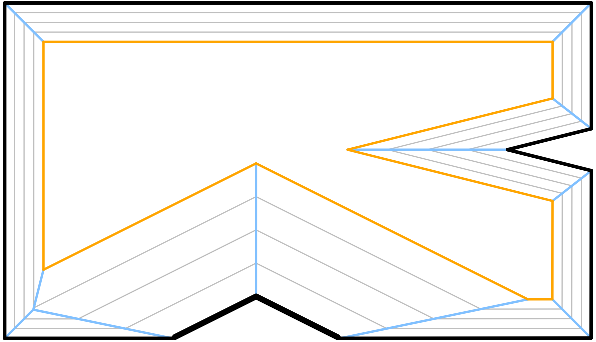

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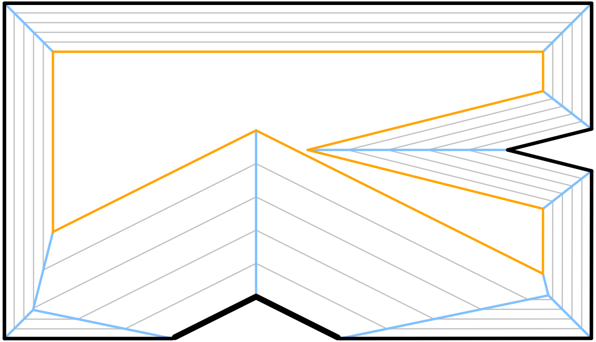

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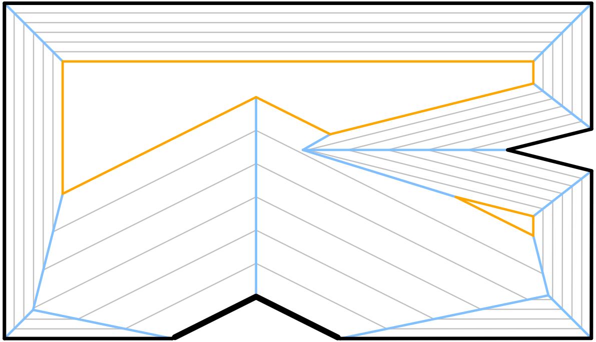

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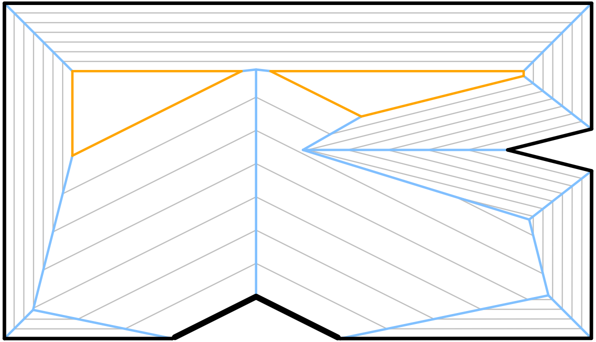

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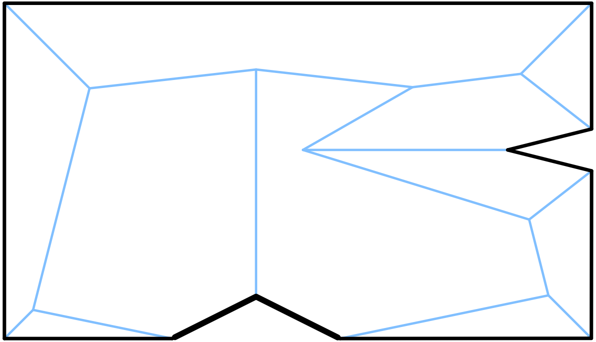

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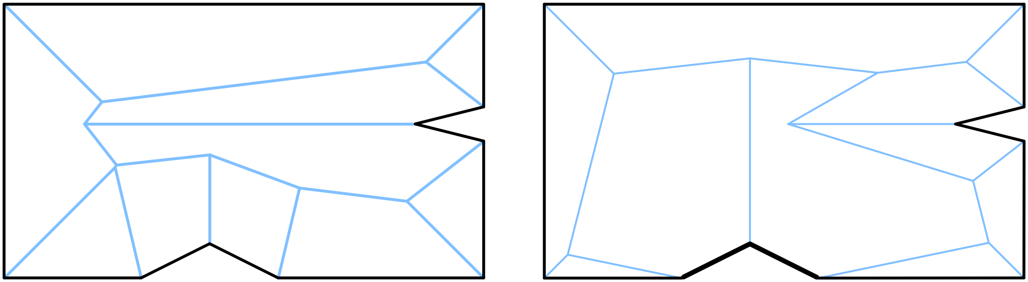

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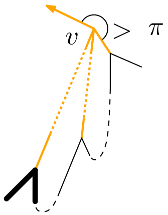

mutli-split event

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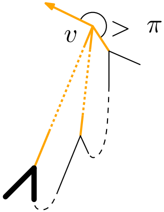

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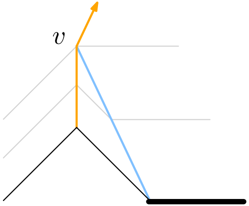

edge event

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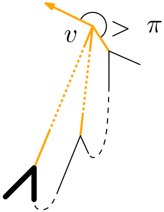

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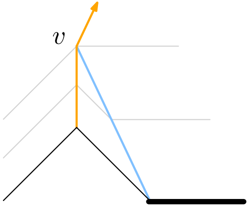

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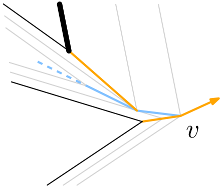

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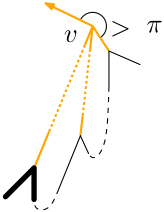

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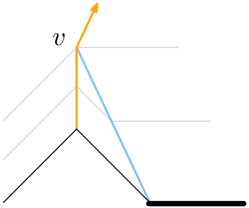

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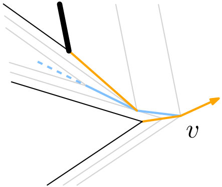

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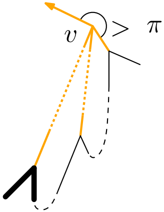

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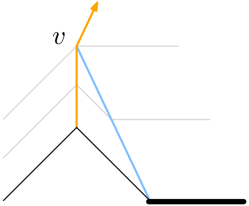

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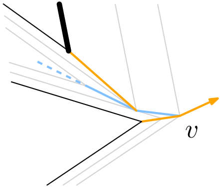

edge event[1]

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Induced Line Arrangement $\mathcal{A}(P)$

- For every reflex vertex v of P we construct a line segment s.
- Let s start at v, lie on $v(t)$ of $\mathcal{W}_P(t, \sigma)$, and end at the first intersection with the boundary of P.
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Linear or Quadratic Space

- Store all $\mathcal{O}(r^2)$ intersections in a sorted manner: $\mathcal{O}(n^2 \log n)$ time and $\mathcal{O}(r^2)$ space. Obtain the next intersection in $\mathcal{O}(1)$ time.
- Store only the closest intersection to a point for all segments in $\mathcal{A}(P)$, i.e., overall $\mathcal{O}(r)$ intersections. Obtain the next intersection in $\mathcal{O}(r)$ time.

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- For every reflex vertex v of P we construct a line segment s.
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- For a fixed k in $1 \leq k \leq r$. Let s in $\mathcal{A}(P)$ and let p the next intersection point on s.
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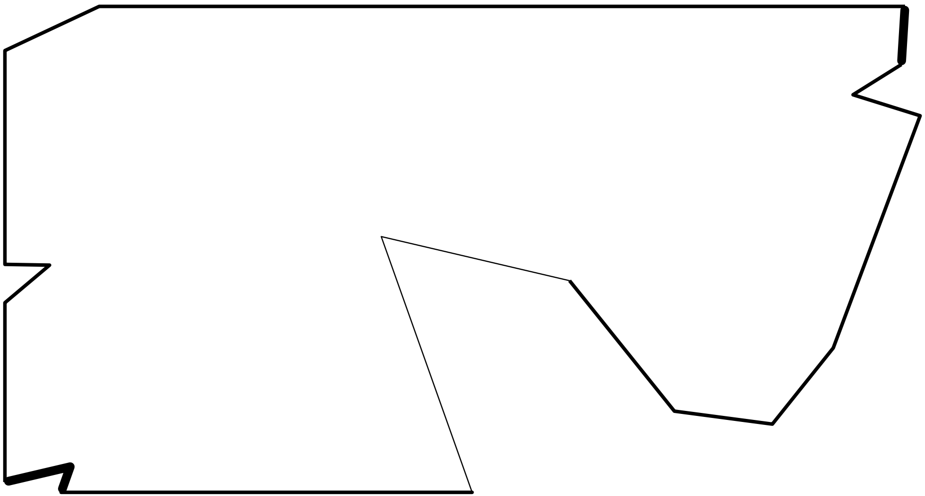

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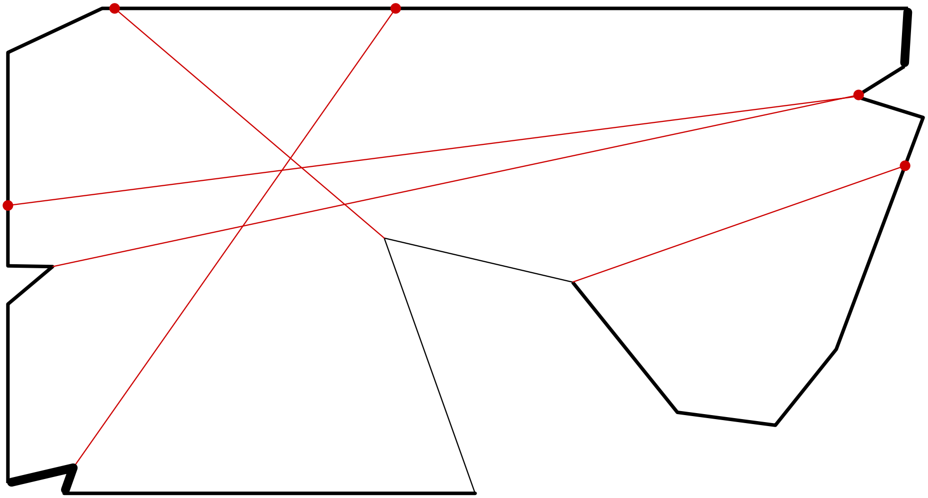

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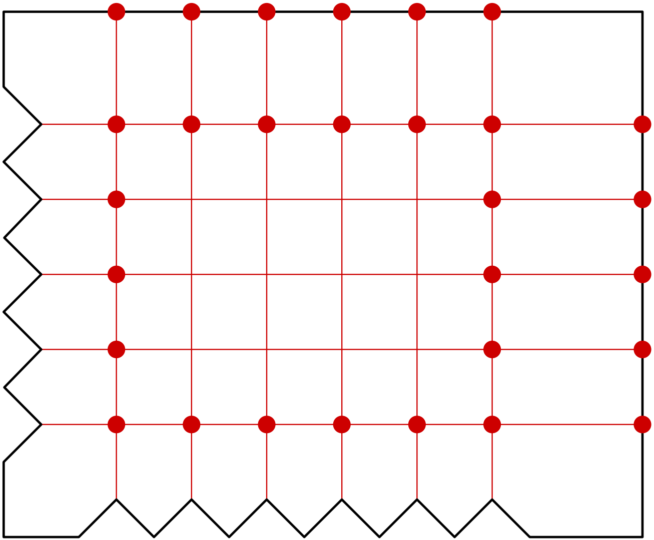

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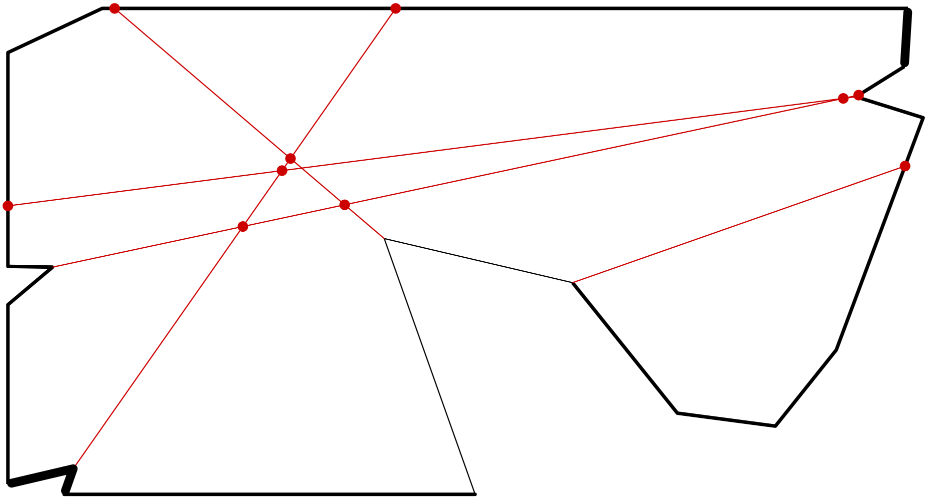

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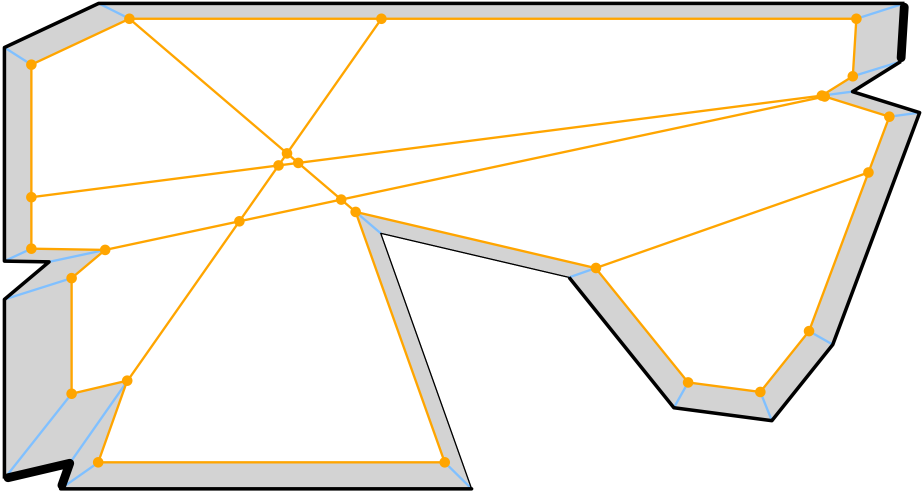

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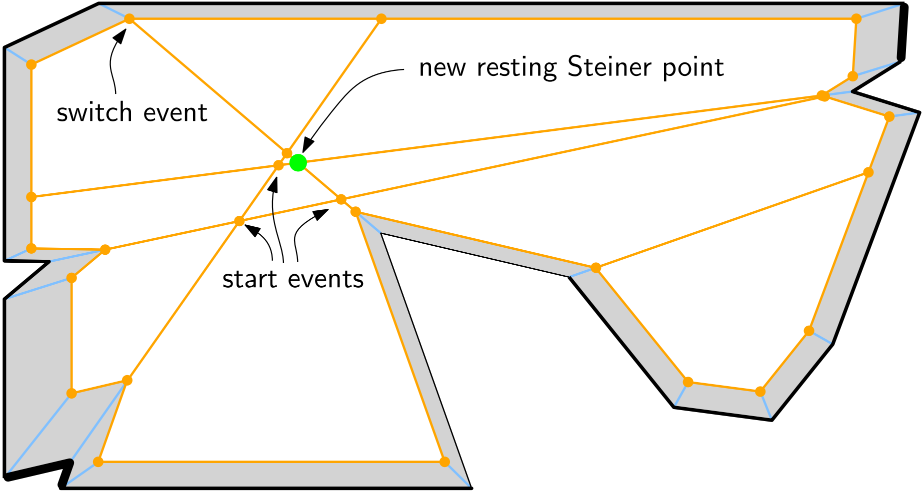

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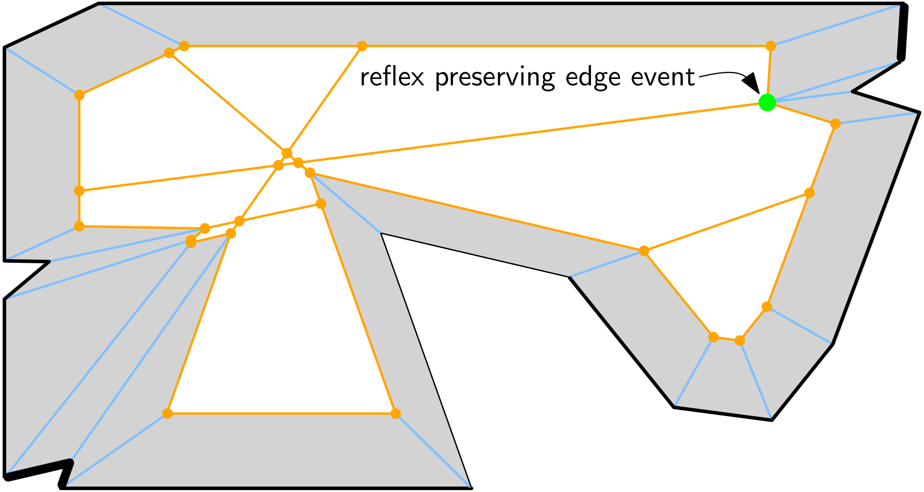

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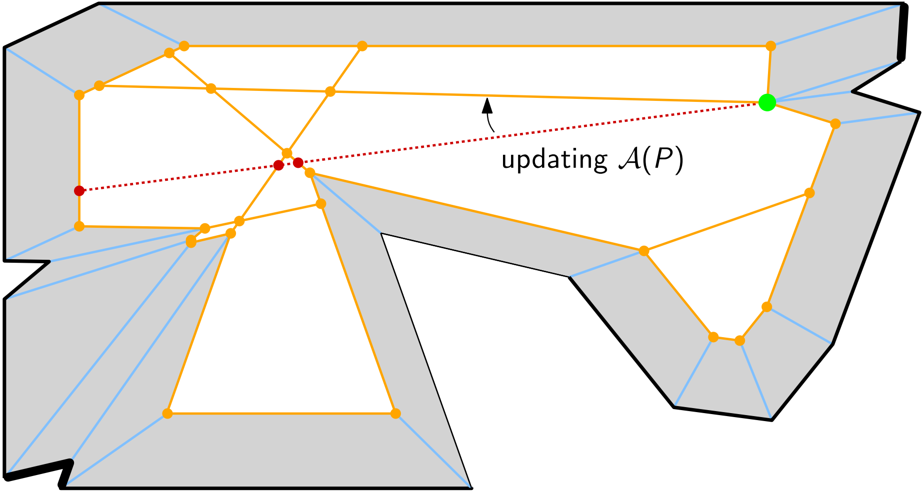

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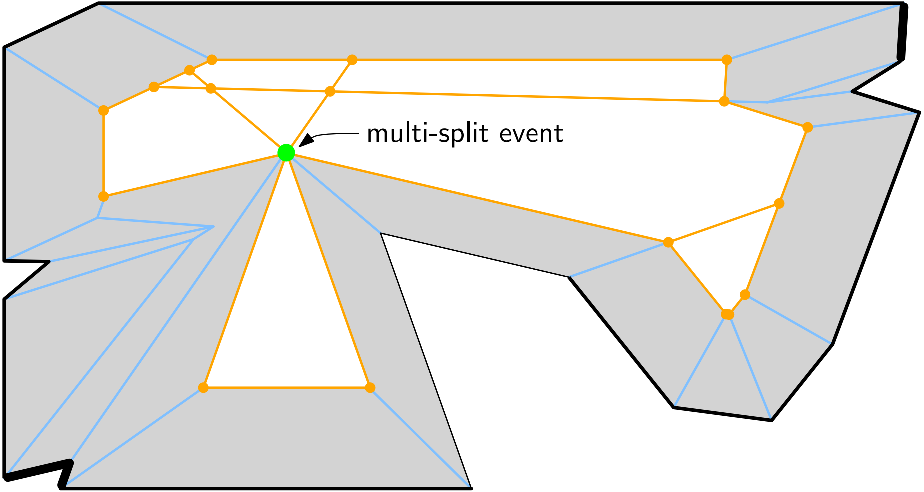

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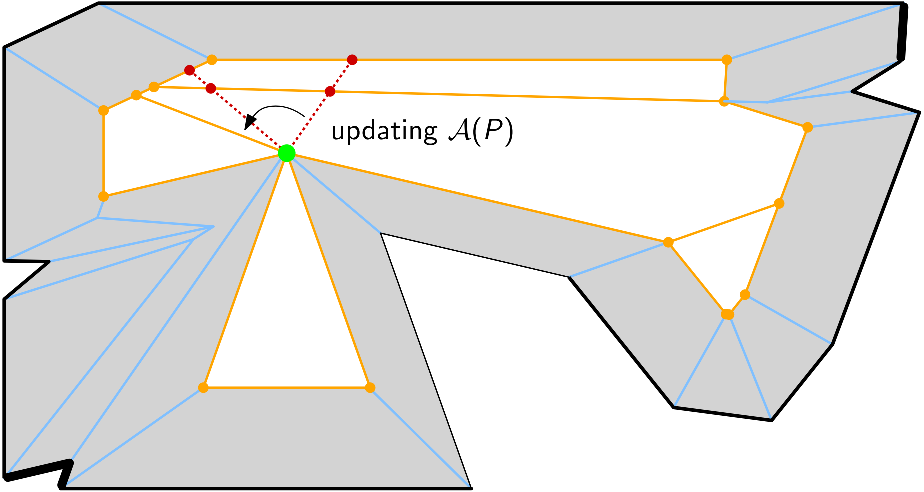

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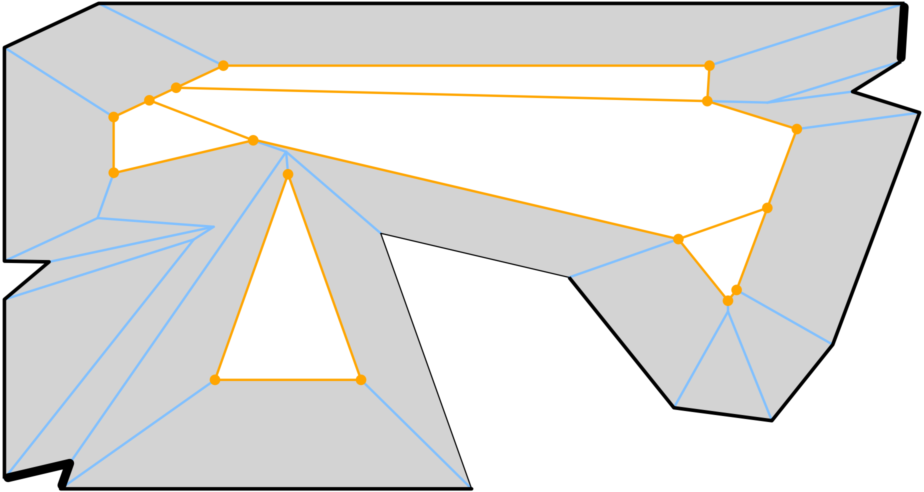

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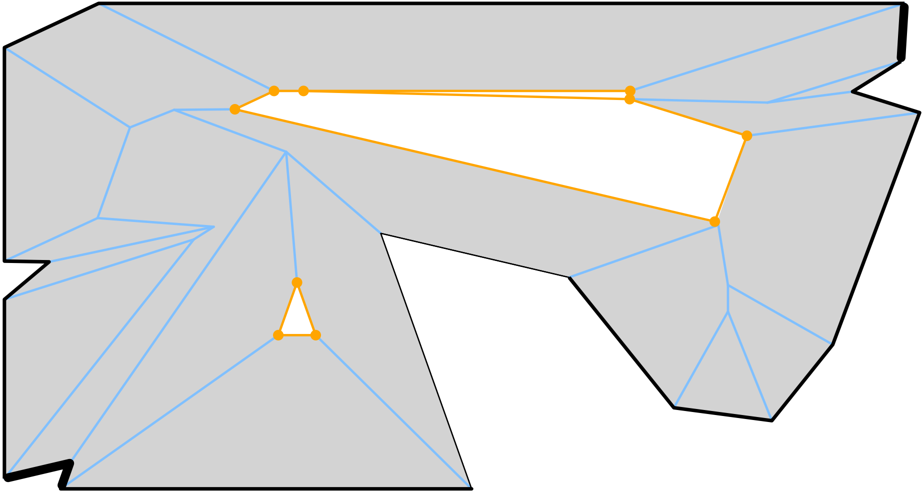

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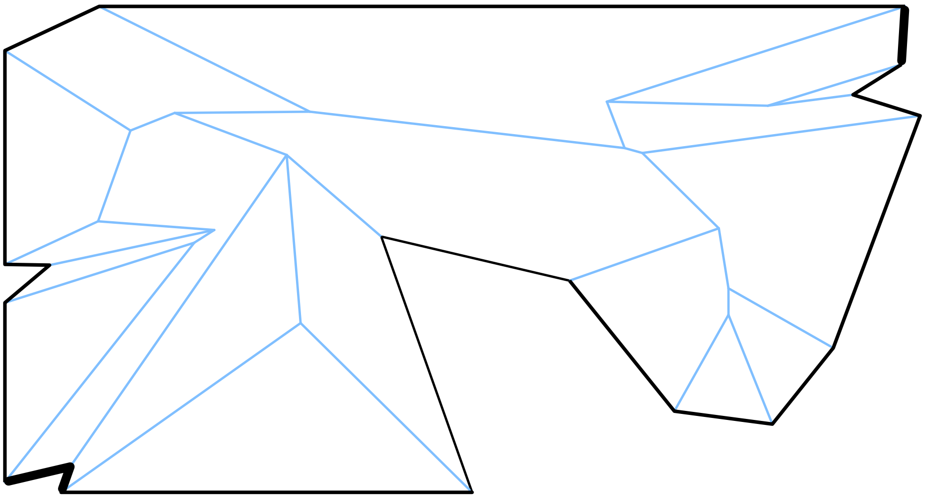

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P consists of n vertices, r of which are reflex. The propagation of $\mathcal{W}_P^*(t, \sigma)$ results in:

- $\mathcal{O}(n)$ split/edge events,
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	time	*space*
classical	$\mathcal{O}(n^2 + r^3 + nr \log n)$	$\mathcal{O}(n)$
trade-off [5]	$\mathcal{O}(n^2 + r^3/k + nr \log n)$	$\mathcal{O}(n + kr)$

[5] with a fixed k s.t. $1 \leq k \leq r$.

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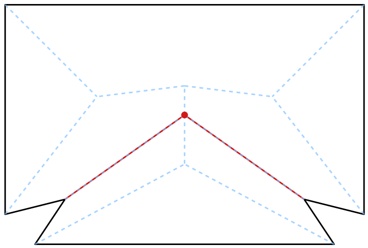

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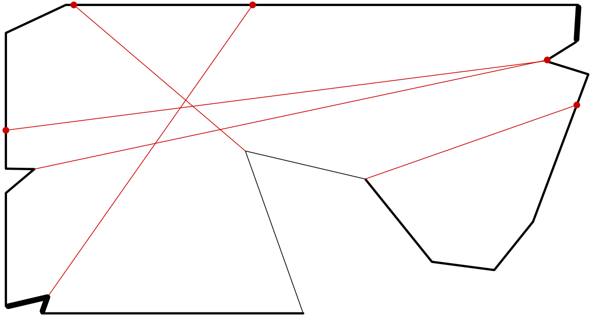

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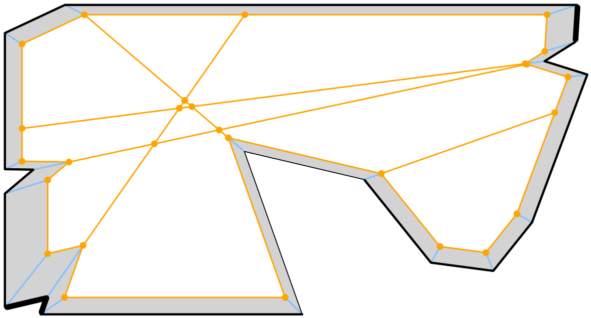

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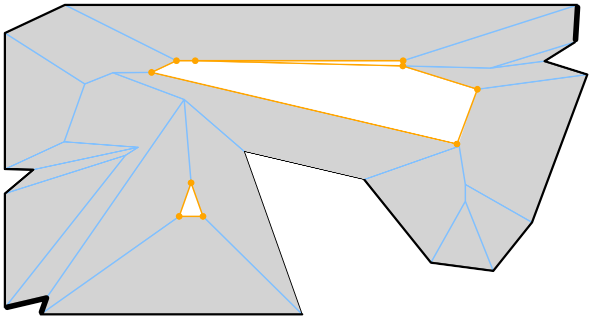

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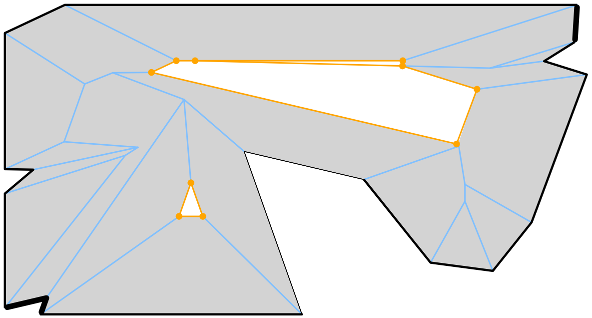

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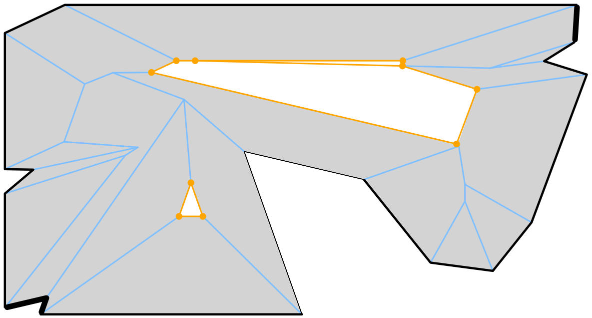

Questions?

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